Indian Institute of Technology, Humanities and Social Sciences Department,

Selection Test for PhD Admissions, December 2018

Total Marks: 100; Duration: 2 hours

Economics

There are two sections in this question paper. Answer <u>All</u> questions in Section A and <u>any four</u> questions in Section B

<u>Section A</u>

1. Which one of the following measures are unaffected by outliers? (2 marks)

   (a) Mean

   (b) Interquartile range

   (c) Standard deviation

   (d) Range

   (e) None of the above

2. Let X be a random variable having the standard normal distribution. If $Y = 2X$, what is the variance of Y? (2 marks)

   (a) 4

   (b) 2

   (c) 16

   (d) 1

   (e) None of the above

3. An estimator of a parameter, θ, is consistent if (2 marks)

   (a) It has low variance

   (b) It is biased

   (c) It takes on the same value in every sample

   (d) It has low mean squared error

(e) None of the above

4. Since the population size is always larger than the sample size, the sample statistic   (2 marks)

   (a) can never be larger than the population parameter

   (b) can never be smaller than the population parameter

   (c) can never be equal to the population parameter

   (d) can never be zero

   (e) None of the above

5. An estimator is   (2 marks)

   (a) a distribution

   (b) a random variable

   (c) also known as estimate

   (d) a non-random number

   (e) None of the above

6. Which statement is correct for the function $f(x) = x^n$ where n an integer?   (2 marks)

   (a) If n is odd and positive then $x = 0$ is a peak

   (b) If n is odd then the function is concave on the whole real line

   (c) If n is odd and negative then the function is concave on the negative numbers

   (d) If n is even then the function is convex on the whole real line

   (e) None of the above

7. Is $x = 0$ an inflection point of the function $y = x^5 + x^3$?   (2 marks)

8. True or false: consider standard utility maximization problem. Price vector is orthogonal to the vector connecting two commodity-bundles on budget hyperplane. Explain.   (4 marks)

9. True or false: let X and Y be finite-dimensional Euclidean spaces and let $\psi : X \rightarrow\rightarrow Y$ be a correspondence. We say that ψ is continuous at any $x \in X$ if it is both upper-hemicontinuous and lower-hemicontinuous at this point. Explain (2 marks)

Section B

Q1 [20 marks].

(a) State if the following statements are true or false. Justify your answers.

    1. Indirect utility function is homogeneous of degree zero in its arguments. (3 marks)

    2. Substitution matrix $S(p, M)^1$ is symmetric and negative semidefinite. (4 marks)

    3. In second-degree price discrimination the monopolist can extract the full consumer surplus. (3 marks)

(b) Two roommates need to clean their apartment and they need to decide on an amount of time each of them would spend on cleaning: (player i spends $t_i \geq 0$). Player i's payoff is given by $(10 - t_j)t_i - t_i^2$ (This payoff function implies that the more one roommate cleans, the less valuable is cleaning for the other roommate.)

    1. What is the best response correspondence of each player? (2 marks)

    2. Which choices survive one round of iterated elimination of strictly dominated strategies? (4 marks)

    3. Which choices survive iterated elimination of strictly dominated strategies? (4 marks)

Q2 [20 marks].

Consider that India has the following production function:

$$Y_t = F(K_t, N_t) = A_t K_t^\alpha N_t^{1-\alpha}, \quad \text{where } \alpha \in (0, 1).$$

Assume for now that A_t is constant over time (there is no technological progress in this economy, so $A_t = A$), g_N is the growth rate of N (worker), δ is the rate of depreciation of capital in this economy, and s is the saving rate.

1. Rewrite the production function in terms of only capital per worker (k_t) and output per worker (y_t). (2 marks)

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[1] As defined in standard consumer behavior analysis

2. Solve for the steady state values of capital per worker (k^*), output per worker (y^*), and consumption per worker (c^*). Draw a diagram that shows all three steady state values you calculated. (4 marks)

3. Find the saving rate at which steady-state consumption is maximized (i.e. we are at the Golden Rule steady state). (2 marks)

4. Suppose that at time t there is a one-time inflow of foreign workers into the country, so that N jumps from N_0 to N_1. (Assume that this does not affect g_N.) Draw two diagrams: one showing what happens to the 'investment' (which is the concave curve) and 'required investment' (which is the straight line from origin) schedules, including dynamics, and one depicting the effects of this inflow on capital per worker over time. (You need to explain the outcome with intuition). (2 marks)

5. Suppose India (B) and Greece (G) have identical production functions and same δ, g_N, and s. However, technology is such that $A_B > A_G$. Which country will have a higher steady-state capital per worker? Prove your answer mathematically and with a diagram. (3 marks)

6. Assume that all countries are heading towards the same steady state (that is, in the long run, all countries have access to the same technology and have the same preferences as manifested in the same saving rate and population growth rate). Does the model predict growth for poorer countries should be faster, slower, or the same as richer countries? Show mathematically. (Hint: Define the growth rate of capital as $g_k = \frac{\Delta k_t}{k_t}$). (3 marks)

7. Suppose that $\alpha = 0.5$ in the given production function (that you solved in part 1). Assume that the level of technology in the country depends on capital per worker, in particular, $A = k^\beta$. Discuss convergence and growth in an economy with $\beta = 0.5$ and compare it to an economy with $\beta < 0.5$. Use diagrams and words. (4 marks)

Q3 [20 **marks**].
You are interested in estimating production function for a set of 29 Indian firms. You regress the log of output (Y) on log of labour (L) and log of capital (K). The following results are obtained.

$$\hat{logY} = 4 + 0.4logL + 0.9logK, \quad R^2 = 8/10, \sum_{i=1}^{29} e_i^2 = 520$$

and

$$X'X = \begin{bmatrix} 29 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & 80 \end{bmatrix}$$

where X denotes the matrix of explanatory variables including the intercept and e_i denotes the residual.

1. Interpret the coefficients of $logL$ and $logK$. (4)

2. Calculate the appropriate test statistic to test the following hypotheses: (i) the coefficient of $logL$

is equal to zero, (ii) the production function is homogeneous of degree 1 in labour and capital, (iii) the overall significance of the model. (4+8+4)

Q4 [**20 marks**].

Consider the following statistics: In 2011, population in the age group of $0-14$ years as proportions of total population were 30.8 per cent for Kerala and 35.8 per cent for Uttar Pradesh. At the same time, the proportions (in total population) of population in the 60+ age group were 12.7 per cent for Kerala and 8.5 per cent for Uttar Pradesh. What do these numbers signify for the future development potential of these States and for India as a whole in the context of the debates on 'demographic dividend' (you can think and write about the human development challenge, employment challenge, the skill challenge, and other related issues).

Q5 [**20 marks**].

As you know, there have been large fluctuations in India's exchange rate in recent years. For instance, the Rupee-Dollar exchange rate was Rs.68 in January 2017, Rs.63 in January 2018 and again Rs.68 in July 2018. What do you think are the reasons for such fluctuations in the exchange rate? How do such fluctuations affect India's industrial and overall economic growth? What do you think has been the impact of oil price movements on all these and on India's overall economic growth prospects?

THE END